Dvali-Gabadadze-Porrati brane as a gravity conductor

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I study how the DGP (Dvali-Gabadadze-Porrati) brane affects particle dynamics in the linearized approximation. I find that once the particle is removed from the brane it is repelled to the bulk. Assuming that the cutoff for the gravitational interaction is $M_* \sim 1/\epsilon$, I calculate the classical self-energy of a particle as the function of its position. Since the particle wants to go to the region where its self-energy is lower, it is repelled from the brane to the bulk where it gains its 5D self-energy. Cases when the mass of the particle $m < 8\pi^2 M_*$ and $m > 8\pi^2 M_*$ are qualitatively different, and in the latter case, one has to take into account the effects of strong gravity. In both cases the particle is repelled from the brane. For $m < 8\pi^2 M_*$ I obtain the same result from the "electrostatic" analogue of the theory. In that language the mass (charge) in the bulk induces a charge distribution on the brane which shields the other side of the brane and provides a repulsive force. The DGP brane acts as a conducting plane in electrostatics (keeping in mind that in gravity different charges repel). The repulsive nature of the brane requires a certain localization mechanism. When the particle overcomes the localizing potential it rapidly moves to the bulk. Particles of mass $m > 8\pi^2 M_*$ form a black hole within $1/M_*$ distance from the brane.

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I. INTRODUCTION

A phenomenologically acceptable five-dimensional brane world theory with one infinite extra dimension was recently developed in [1–3]. The low scale of quantum gravity M_* is pulled (renormalized) at the brane by the high scale M_{SM} that describes the brane localized standard model. More precisely, the 4D Einstein Hilbert term, with a strength $\sim M_{\mathrm{P}}^2 \sim M_{SM}^2$, is induced on the brane. This effect ensures that the observer on the brane sees weak 4D gravity (Newton constant $G_N \sim 1/M_{\mathrm{P}}^2$) up to the distance $r_c = M_{\mathrm{P}}^2/M_*^3$. At distances bigger than r_c gravity becomes five dimensional. At short distances gravity is modified by quantum corrections at M_*^{-1}. Short distance gravity measurements exclude the modification of the laws of gravity at distances bigger than ~ 0.1 mm$\sim 1/10^{-3}$ eV [4]. Cosmological observations on the other hand suggest that gravity is not changed to distances of order $\sim 10^{29}$ mm. Thus the present knowledge about gravity constrains the scale of gravity in this class of models to the range

$$10^{-3}\ \mathrm{eV} < M_* < 10\ \mathrm{MeV}. \qquad (1.1)$$

Relativistic corrections and the question of how are they encoded in the tensor structure of the graviton propagator were studied in [5]. The cosmological consequences of the model and especially the fact that the model gives rise to an accelerated universe (as observed in [6]) were considered in [7]. The relevance for the solution to the cosmological constant problem was considered in [8]

In the present paper I study how the induced 4D Einstein Hilbert term affects the dynamics of the particle of mass m at a distance y_0 from the brane. This question is important in order to identify the experimental signatures of collider black hole production in this class of models. Since the particle itself induces the metric, there is no static metric in which one could study the behavior of geodesic lines (that describe the motion of the test particle). Instead of trying to solve the problem in full relativistic theory, we will limit ourselves to the Newtonian approximation and use some basic facts about black holes. The result that I find is that the brane repels particles into the bulk where they have a lower (bigger in magnitude and negative) self-energy.

In the next section I derive and briefly discuss Newtonian potential. In the third section I find the dependence of self-energy of a particle as a function of its distance from the brane. Once the particle leaves the brane, the gradient of self-energy forces it to go from the brane to the bulk. I also describe the process of black hole formation for particles with mass $m > 8\pi^2 M_*$. In the fourth section I present the "electrostatic" analogue derivation of repulsive force for particles with mass $m < 8\pi^2 M_*$. Finally, in the discussion, I address questions regarding phenomenological consequences of the repulsive nature of the brane.

II. NEWTONIAN POTENTIAL

Action for the model [1] is the sum of the 5D Einstein Hilbert term and the induced 4D term on the brane

$$S = M_*^3 \int d^4x\, dy\, \sqrt{G}\,\mathcal{R}_{(5)} + M_{\mathrm{Pl}}^2 \int d^4x\, \sqrt{|g|}\,R. \qquad (2.1)$$

Here I divide 5D coordinates into a 4D part (Greek indices) and the extra coordinate y like $X^A = (x^\mu, y)$, G_{AB} is 5D metric and $\mathcal{R}_{(5)}$ its curvature scalar, $g_{\mu\nu}(x^\mu) = G_{AB}(x^\mu, y = 0)\,\delta_\mu^A \delta_\nu^B$ is induced 4D metric (I take straight brane located at $y = 0$) and R the corresponding scalar curvature. The tension of the brane is taken to be zero. If we take the limit of slowly varying weak fields, equations of motion reduce to the equations for deviation of the g_{00} component from the

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flat space constant value (scalar gravity). The equation for (Euclidean) Green's function for the scalar gravity case reads

$$(\Box_4[1+r_c\delta(y)]-\partial_y^2)G(x-x_0,y,y_0)=\delta^4(x-x_0)\,\delta(y-y_0), \tag{2.2}$$

which has the solution (see [3])

$$G(p,y,y_0)=\frac{1}{p}e^{-p|y-y_0|}-\frac{1}{p}e^{-p(|y|+|y_0|)}\frac{1}{1+1/r_cp}. \tag{2.3}$$

Let us evaluate the exact Newtonian potential at the point $(\vec{x},y)$ due to a static source of mass m located at the position $(\vec{x}',y_0)$. The potential is given as a Fourier transform of the Green's function (2.3) integrated over the time

$$V(r,y,y_0)=-\frac{m}{16\pi^2M_*^3}\left(\frac{1}{r^2+(y-y_0)^2}-\frac{1}{r^2+(|y|+|y_0|)^2}-\frac{i}{2rr_c}e^{(|y|+|y_0|-ir)/r_c}\{\Gamma_0[(|y|+|y_0|-ir)/r_c]-e^{2ir/r_c}\Gamma_0[(|y|+|y_0|+ir)/r_c)]\right\}. \tag{2.4}$$

Here $r=|\vec{x}-\vec{x}'|$ and $\Gamma_0(z)$ is an incomplete gamma function (see the Appendix). The potential (2.4) can be expanded in powers of $1/r_c$:

$$V^1=-\frac{m}{16\pi^2M_*^3}\frac{1}{rr_c}\arctan\frac{r}{|y|+|y_0|}, \tag{2.5}$$

$$V^2=-\frac{m}{16\pi^2M_*^3}\frac{1}{rr_c}\left[\frac{(|y|+|y_0|)}{r_c}\arctan\frac{r}{|y|+|y_0|}+\frac{1}{2}\left(\frac{r}{r_c}\right)\ln\frac{r^2+(|y|+|y_0|)^2}{r_c^2}-(1-\gamma)\left(\frac{r}{r_c}\right)\right], \tag{2.6}$$

where $\gamma\approx 0.5772$ is Euler's constant and superscripts on potential denote terms in expansion. The potential to first order in $1/r_c$ was discussed in detail in [3]. Let me briefly discuss potential (2.4). If the mass is on the brane ($y_0=0$), the first two terms in Eq. (2.4) cancel. The potential on the brane $(r,y=0)$, at distances $r\ll r_c$, is four dimensional and the Newton's constant is $G=1/(32\pi M_*^3 r_c)$. As r/r_c increases towards one, the second term in Eq. (2.6) weakens its strength. Finally, for $r\gg r_c$ ($pr_c\ll 1$) the potential becomes purely five dimensional [the first term in Eqs. (2.3) and (2.4)]. Similar behavior occurs if one looks at the potential at $(r=0,y)$. For small y it is a weak four-dimensional potential with constant $2G/\pi$. For $y/r_c\gg 1$ one can find the form of the potential by expanding Eq. (2.4) (see the Appendix) and again obtain the expected five-dimensional behavior. Up to a constant, the potentials have the following short distance expansion and asymptotic behavior:

$$V(y=0,r\ll r_c)=-\frac{m}{32\pi M_*^3}\frac{1}{rr_c}\left(1+\frac{2}{\pi}\frac{r}{r_c}\ln\frac{r}{r_c}\right),$$

$$V(y=0,r\gg r_c)=-\frac{m}{16\pi^2M_*^3r^2}, \tag{2.7}$$

$$V(r=0,y\ll r_c)=-\frac{m}{16\pi^2M_*^3}\frac{1}{|y|r_c}\left(1+\frac{y}{r_c}\ln\frac{y}{r_c}\right),$$

$$V(r=0,y\gg r_c)=-\frac{m}{16\pi^2M_*^3|y|^2}. \tag{2.8}$$

Similar expansions can be easily obtained for a potential at any "angle" in the $r-y$ plane.

If the mass is in the bulk we have two different cases. For particles on opposite sides of the brane, the first two terms in Eq. (2.4) cancel and particles interact via the weak four-dimensional gravity at distances $\sqrt{(|y|+|y_0|)^2+r^2}\ll r_c$. That means that the brane is shielding one side of the brane from the five-dimensional gravitation of sources on the other side of the brane. The effective radius of shielding is $\sim r_c$. If the sources are on the same side of the brane, the interaction is dominated by the first two terms in Eq. (2.4). Masses, sufficiently far from brane, basically interact via strong five-dimensional gravity.

One can illustrate this behavior by plotting the contours of the constant potential of the body as it moves from the bulk towards the brane (Fig. 1). At Schwarzschild radius $g_{00}\approx(1+2V)$ diverges. Although I do not have a relativistic solution to the system, one would expect that the Schwarzschild surfaces (black hole horizons) behave as surfaces of constant potential $V\approx -1/2$.

III. SELF-ENERGY

In this section I will evaluate the classical self-energy of the particle of mass m in the presence of the Dvali-Gabadadze-Porrati (DGP) brane. My main assumption, along the lines of [3], is that the gravity is cut off at distances $\epsilon\sim 1/M_*$. Classically, gravitational self-energy is determined by the cutoff distance and the form of the potential. Since the potential changes with the position, the gravitational self-energy of a particle will be a function of its distance from the brane. Gradient of self-energy will give rise to a force that will try to move the particle to the region of lowest gravitational self-energy. For $m<8\pi^2M_*$ I will use Newtonian approximation, since the potential is weak at the cutoff distance $\sim M_*^{-1}$. For $m>8\pi^2M_*$, I will use a Newton-like approxi-

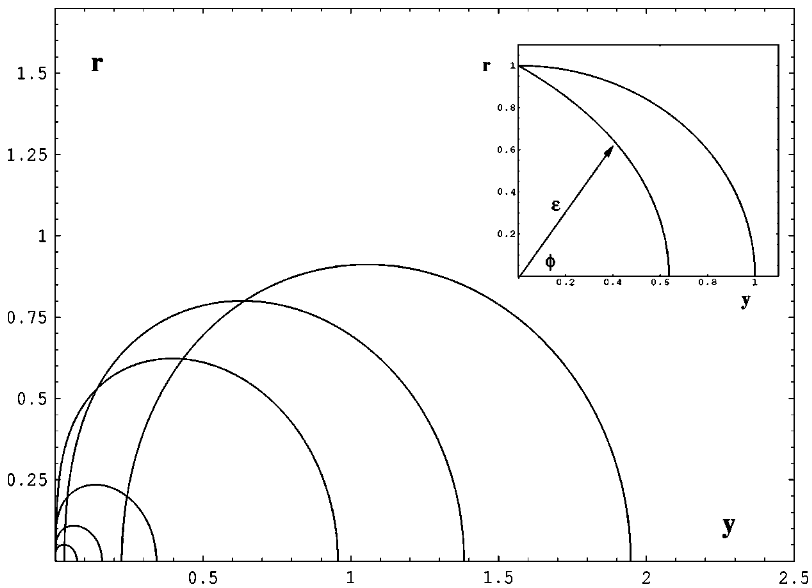

FIG. 1. Surfaces of constant potential $V = -(1/16\pi^2)m/M_*$. Point mass is on $r=0$, $y_0 = 1, 0.5, 0.2, 10^{-2}, 10^{-3}, 10^{-4}$ in units of M_*^{-1}. Inset: anisotropic cutoff distance ϵ for a mass on the brane.

mation, which will incorporate some basic facts of general relativity.

A. Case $m < 8\pi^2 M_*$

For $m < 8\pi^2 M_*$ Schwarzschild radius in the bulk $r_{S5} = (1/M_*)\sqrt{m/8\pi^2 M_*}$ [9] is smaller than the cutoff M_*^{-1} so Newtonian approximation is justified. Let me remind how we calculate gravitational self-energy of a particle. One wants to find an energy gained by assembling a particle of mass m. Equivalently, one can find the energy needed to destroy a particle by taking away infinitesimal pieces of matter and removing them to infinity, where the potential is defined to be zero. If the force starts to act at a distance ϵ from the center-of-mass distribution of a particle, one then finds the expression for self-energy to be

$$W = -\int_0^1 (1-M)\,dM \int_\epsilon^\infty \frac{dV(r)}{dr}dr = \frac{1}{2}V(\epsilon). \tag{3.1}$$

In my normalization of the Newton constant, that leads to self-energies in four-dimensional theory (on the brane) and five-dimensional theory (infinitely far from the brane),

$$W_4 = -\frac{1}{64\pi}\left(\frac{m}{M_\mathrm{P}}\right)^2\frac{1}{\epsilon}, \quad W_5 = -\frac{1}{32\pi^2}\left(\frac{m^2}{M_*^3}\right)\frac{1}{\epsilon^2}. \tag{3.2}$$

If we take that the gravity cutoff ϵ is just an inverse scale of gravity M_*, then the ratio of self-energies in pure 4D and 5D theories is

$$\frac{W_4}{W_5} = \frac{\pi}{2}\left(\frac{M_*}{M_\mathrm{P}}\right)^2. \tag{3.3}$$

In pure four- or five-dimensional theories energy of the mass density $\rho(x)$ is given by $W = (1/2)\int \rho(x)V(x)d^n x$. One can then use Gauss' law to relate mass density and divergence of the field. After partial integration energy can be written as an integral over the square of the field $W \sim -\int |\nabla V(x)|^2 d^n x$. By integrating energy stored in the field in pure 4D and 5D theories one again finds expressions (3.2). I must stress here that the discussed theory is neither purely 4D or 5D theory. In particular, for the mass on the brane, Gauss' law is not valid (if I call r_5 the 5D radial distance from the mass at $y_0 = 0$, then the field drops $\sim 1/r_5^2$, while the surface area increases $\sim r_5^3$). For this reason I will use Eq. (3.1) when calculating self-energy of the mass at an arbitrary position in 5D space (one should not use Gauss' law).

Let us look at the self-energy of the particle on the brane. By using prescription (3.1) we find that the self-energy is that of a four-dimensional particle. However, since our space is not isotropic there is an apparent ambiguity in self-energy. It depends on the direction from which we assembled the particle. Let us define polar coordinates $\rho = r^2 + y^2$ and $\phi = \arctan(r/y)$ and assemble the particle at $y_0 = 0, r = 0$ by bringing infinitesimal masses from infinity and direction ϕ from the bulk. Self-energy of the particle on the brane then varies by a factor $\pi/2$ (same as the Newton constant) for angles $0 < \phi < \pi/2$:

$$W(y_0 = 0) = -\frac{1}{32\pi^2}\left(\frac{m}{M_\mathrm{P}}\right)^2\frac{\phi}{\sin\phi}\frac{1}{\epsilon}. \tag{3.4}$$

Since the self-energy must be a well-defined quantity, I conclude that the factor $\phi/\sin\phi$ defines the physical cutoff distance when the mass is on the brane. Certainly, the space in question is not isotropic, and I cannot assume that the gravity cutoff surface is a 3-sphere, but rather a surface of constant field strength with average distance from the particle $\sim 1/M_*$ (inset to Fig. 1). For the arbitrary position of particle y_0, the expression for self-energy to first order in $1/r_c$ is

$$W(y_0)=-\frac{1}{32\pi^2}\left(\frac{m^2}{M_*^3}\right)\times\left[\frac{1}{\epsilon^2}\left(1-\frac{1}{1+4(y_0/\epsilon)^2+4(y_0/\epsilon)\cos\phi}\right)+\frac{1}{r_c\epsilon}\frac{1}{\sin\phi}\arctan\left(\frac{\epsilon\sin\phi}{2y_0+\epsilon\cos\phi}\right)\right]. \quad (3.5)$$

For $y_0>\epsilon$, dependence on ϕ can be neglected and the self-energy has the form

$$W(y_0)=W_5\left(1-\frac{1}{1+4(y_0/\epsilon)^2}\right)+W_4\frac{1}{\pi(y_0/\epsilon)}. \quad (3.6)$$

For $0<y_0<\epsilon$ (neglecting the small $1/r_c$ contribution of 4D self-energy), one gets the same answer by following argument. The first term in Eq. (3.5) represents isotropic 5D interaction and should be cut off at the surface of three sphere of radius ϵ. The second term is an anisotropic contribution and its cutoff has to be defined so that it does not depend on the angle ϕ. If I define the (angle) dependent cutoff for the second term as $\tilde{\epsilon}^2=\epsilon^2 f(y_0/\epsilon,\phi)$ one finds that $f(y_0/\epsilon,\phi)=[-2(y_0/\epsilon)\cos\phi+\sqrt{1+4(y_0/\epsilon)^2\cos^2\phi}]$. Plugging this back to Eq. (3.5) one obtains the behavior of the 5D contribution as in Eq. (3.6).

To summarize, at the brane, the particle has 4D self-energy, upon leaving the brane, within a couple of ϵ distance, it gains the biggest part of its 5D self-energy and looses its 4D self-energy. The particle at $y_0>\epsilon$ will feel strong force

$$F_y(y_0)=-\frac{dW(y_0)}{dy_0}=\frac{1}{64\pi^2}\frac{m^2}{M_*^3}\frac{1}{y_0^3}\left(1-\frac{y_0}{r_c}\right). \quad (3.7)$$

This force will try to push the particle to the bulk where its self-energy increases in magnitude by the large factor of $(M_{\rm P}/M_*)^2$.

B. Case $m>8\pi^2 M_*$

If the mass of the particle is bigger than the scale of gravity M_*, I cannot calculate the self-energy by cutting off the Newtonian potential at $\epsilon\sim M_*^{-1}$. The reason is that the Schwarzschild radius in the bulk is bigger than the inverse scale M_*, and at distances shorter than Schwarzschild radius r_{S5} gravity is not weak. In following considerations I will not write negligible corrections of the order of $1/r_c$.

Let us calculate the self-energy of a 5D black hole by approximating the black hole as an object that gravitates via Newton's law at distances bigger than the event horizon $r>r_{S5}$. What happens with potential at distances below the event horizon does not influence the energy of the world outside the horizon. Again I construct the self-energy by assembling the black hole at the origin out of infinitesimal pieces of matter located at infinity. To assemble the point mass M_* I need to bring matter to a cutoff distance M_*^{-1}, since the Schwarzschild radius is smaller then the inverse cutoff. This contribution is equal to $W_5(m=M_*)=-M_*/(32\pi^2)$. Work gained in bringing the rest of the mass (from M_* to m) would be the work done in bringing it only to the Schwarzschild radius. Outside the horizon there is no change in energy no matter how the potential changes below the horizon. Now I introduce a loose definition of the Schwarzschild radius as a radius at which Newtonian potential has the value $-1/2$. Since the potential energy gained by bringing the mass dM from infinity to Schwarzschild radius is by (our) definition $dM/2$, the total self-energy of a 5D black hole of mass m is

$$W(m>M_*,y_0\to\infty)=W_5(m=M_*)-\frac{1}{2}\int_{M_*}^{m}dM=-\frac{1}{32\pi^2}M_*-\frac{1}{2}(m-M_*). \quad (3.8)$$

In our simplified model of a black hole, self-energy is negative and of the order of (factor of 1/2) the mass of a black hole. It is interesting to note that self-energy could, in principle, be equal to the rest mass so that it would not cost anything to produce it. The situation is reminiscent of the fact that the total mass of the universe, Newton's constant, and the Hubble radius conspire in such a way that it might not cost anything to create particles at the center of the universe, since their rest mass energy is of the order of their gravitational (negative) energy.

Let us see how the formation of the black hole happens as I remove a particle of mass $m>8\pi^2M_*$ from the brane. On the brane, the Schwarzschild radius $r_{S4}\sim M_{\rm Pl}^{-1}(m/M_{\rm Pl})$ is much smaller than the cutoff distance M_*^{-1}. For that reason the self energy on the brane is given by W_4 [Eq. (3.2)]. The self-energy on the brane is much smaller than the self-energy far away from the brane [Eq. (3.8)] and can be neglected. Thus particles of mass $m>8\pi^2M_*$ will (as well as particles with $m<8\pi^2M_*$) be repelled from the brane to the bulk where their self-energy is lower. However, the character of the repelling force will differ from the case of $m<8\pi^2M_*$. Let me define the Schwarzschild surface as a surface on which $V(r,y,y_0)=-1/2$. When the particle is removed from the brane, the Schwarzschild surface expands from a point [actually, the three sphere of radius $r_{S4}\sim M_{\rm Pl}^{-1}(m/M_{\rm Pl})$] anisotropically (Fig. 1). After the particle reaches a certain value of y_0, the Schwarzschild surface crosses the three sphere of radius M_*^{-1} that describes the gravity cutoff radius (Fig. 2) . This crossing first happens for the value of $\phi=0$. Up to that point the self-energy and the force on particle are the same as in the case $m<8\pi^2M_*$. After that point we cannot consider M_* self-energy cutoff distance and the self energy evolves different from the case $m<8\pi^2M_*$. Moving the particle further into bulk, the Schwarzschild surface grows and takes over the M_* sphere at larger angles and finally, for some critical value of y_0, the M_* sphere becomes completely contained inside the Schwarzschild surface. At that point we can say that the

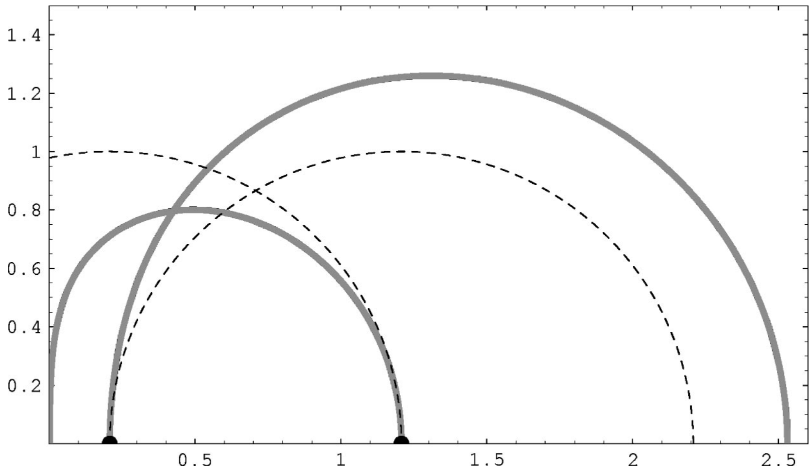

FIG. 2. The formation of the black hole horizon in the $r-y$ plane. The horizontal axis measures distance from the brane y and the vertical axis measures the distance along the brane r, both in units of M_*^{-1}. The mass of the particle is taken to be $m=16\pi^2 M_*$, i.e., twice the critical mass. Solid lines represent Schwarzschild surfaces, dashed lines represent surfaces of three spheres of unit radius, and masses are represented by dots. The horizon emerges when the particle is at $y_0^{\phi=0}=0.207$ and completely encloses the M_* sphere one unit farther.

black hole formation is finished and the particle will have (up to corrections of the order of M_*) the self-energy of a 5D black hole.

Points where the Schwarzschild surface crosses the M_* sphere for $\phi=0,\pi$ can be expressed from potential (2.4)

$$y_0^{\phi=0}=\frac{1}{2M_*}\left(\frac{1}{\sqrt{1-8\pi^2 M_*/m}}-1\right),$$

$$y_0^{\phi=\pi}=y_0^{\phi=0}+\frac{1}{M_*}. \tag{3.9}$$

For $m \gg 8\pi^2 M_*$ these expressions become $y_0^{\phi=0}\approx 2\pi^2/m$ and $y_0^{\phi=\pi}\approx 1/M_*$. Thus the horizon starts forming at $\sim 2\pi^2/m$ and is formed precisely $1/M_*$ farther. The force felt by the particle on $y_0<2\pi^2/m$ is the same as in the case $m<8\pi^2 M_*$ $(y_0 M_* \ll 1)$.

$$F_y(y_0<2\pi^2/m)=\frac{1}{4\pi^2}m^2 M_* y_0. \tag{3.10}$$

For $2\pi^2/m<y_0<1/M_*$, neglecting terms of order M_* in self-energy, force is approximately

$$F_y(y_0>2\pi^2/m)\approx\frac{\pi^2}{y_0^2}. \tag{3.11}$$

To summarize, for $m>8\pi^2 M_*$, the 5D Schwarzschild radius is bigger than the cutoff M_*^{-1}, and one cannot use the Newtonian theory to calculate the self-energy of the particle. Modeling a black hole as an object that gravitates with Newtonian potential outside the horizon, I calculated self-energy and estimated the force felt by the particle. As in the case $m<8\pi^2 M_*$ particles are repelled from the brane.

IV. CONDUCTOR ANALOGY

In this section I will rederive results of the previous section for particles with $m<8\pi^2 M_*$ from a different point of view. The Lagrangian of our theory can be thought of as the Lagrangian for a purely 5D theory with a specific type of source localized at $y=0$. One would expect this correspondence to be valid everywhere except at $y=0$ (world volume of the source) because the source itself is a kinetic term for the 4D theory. At $y=0$, the value of the delta function diverges and the 4D kinetic term becomes dominant. The gravity theory in this approach $(y\neq 0)$ becomes equivalent to a 5D gravity in the presence of an infinite three-plane with a specific mass (charge) distribution. In the Newtonian approximation theory is equivalent to the electrostatic setup of a charge near the conducting plane. I will use the symbol $\vec{E}$ for the gravitational field and sometimes interchange the terms mass and charge.

Let us take mass m at position $(\vec{r}=0,y_0)$ and look at the field at position $(\vec{r},y)$. One can ask what kind of charge distribution on the plane would produce potential (2.4).

A component of the field in the y direction is discontinuous at $y=0$ with discontinuity (to a first order in $1/r_c$)

$$(E_y^{y=+0}-E_y^{y=-0})=\frac{m}{4\pi^2 M_*^3}\,\frac{|y_0|-(r^2+y_0^2)/(2r_c)}{(r^2+y_0^2)^2}. \tag{4.1}$$

Applying the Gauss theorem on the 5D pillbox, as shown in Fig. 3, we can find the charge distribution on the plane

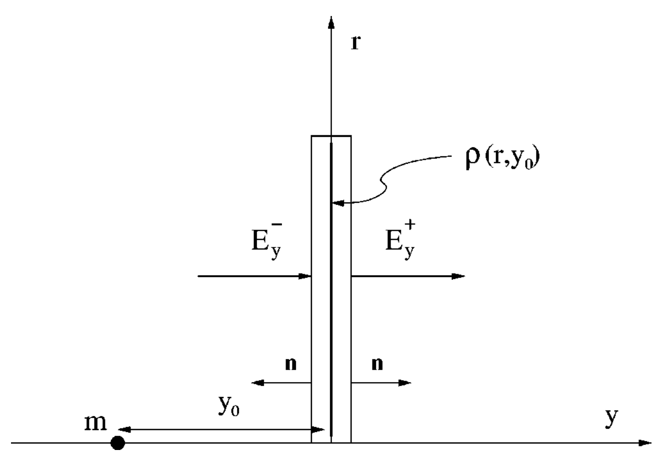

FIG. 3. Gaussian pillbox for the determination of the effective charge distribution $\rho(r,y_0)$ induced by the charge m at position $(r=0,y_0)$.

$$\nabla\cdot\mathbf{E}=-\frac{1}{4M_*^3}\rho(\vec{x},y)\to\rho(r,y_0)$$
$$=-\frac{1}{\pi^2}\frac{m|y_0|}{(r^2+y_0^2)^2}+\frac{m}{2\pi^2 r_c(r^2+y_0^2)}. \qquad (4.2)$$

The first term in the expression for charge density (4.2) represents the distribution of negative charge, sharply peaked around $r=0$, i.e., the projection of the charge position on the brane. Its integral over the volume of the brane is independent of y_0 and equals precisely $-m$. As $y_0\to 0$ this term approaches distribution of a point-like charge $-m\,\delta(\vec{r})$. The second term in Eq. (4.2) is a small (notice $1/r_c$ suppression) distribution of positive charge, much less localized then the negative charge distribution. In order to find the total induced charge density we should integrate the exact expression obtained from the potential (2.4) that is correct to all orders in $1/r_c$. The integral of the charge distribution due to the third term in Eq. (2.4) can be obtained numerically and is equal to m. Thus the total charge induced on the brane due to the charge m in the bulk is zero, as one would expect.

Having the charge distribution, we can calculate the interaction energy between the mass and the induced distribution ("image" distribution of mass $-m$ and the background distribution of mass $+m$). For $y\neq 0$ the theory is just the 5D Newtonian gravity so the potential energy of interaction is (to first order in $1/r_c$)

$$W(y_0)=\frac{1}{2}\int \rho(x)V(x)dV$$
$$=-\frac{m}{32\pi^2 M_*^3}\int\frac{\rho(r,y_0)4\pi r^2 dr}{r^2+y_0^2}$$
$$=\frac{1}{128\pi^2}\left(\frac{m^2}{M_*^3}\right)\frac{1}{y_0^2}\left(1-\frac{2y_0}{r_c}\right). \qquad (4.3)$$

Cutoff effects in this derivation were neglected, so it is understood that $y_0>\epsilon$. The result (4.3) coincides with Eq. (3.6) and gives the same force (3.7).

Let me summarize what happens in our 5D analog picture. Charge m in the bulk induces negative "image" charge distribution of total charge $-m$, localized at the $r=0$, and the small uniform background positive mass distribution. The total induced charge on the brane is zero. As charge m approaches the brane ($y_0\to 0$) image charge distributions tends to the distribution of a point-like charge $-m$, which strongly repels mass m. Finally, charge m and the image $-m$ annihilate and m distributes itself uniformly on the brane. The described process is completely analogous to the behavior of the charge near the conducting plane. The only difference is that in electrostatics, charges of the opposite sign attract and in gravity they repel. Thus mass m in the bulk is repelled from the brane by its image $-m$. In this sense the DGP brane acts as a gravity conductor, shielding the fields and giving rise to a repulsive force. One could imagine constructing tensionless objects with this property that would gravitationally repel masses or act as gravitational dipoles. In cosmological setups the tensionless DGP brane would gravitationally shield ("shadow") parts of the universe and could modify cosmological evolution.

V. DISCUSSION

In previous sections I showed that the particles are repelled from the brane that induces the kinetic term. By analogy with an ordinary wall with a tension [10], we can loosely say that the induced kinetic term creates localized energy momentum density on the brain in which repulsive tension dominates over attractive energy density. Theories with a low scale of gravity predict collider production of black holes. Because of the repulsive nature of the brane, black holes produced in collider experiments would be repelled to the bulk.

The repulsive nature of the brane requires a certain localization mechanism for standard model particles. We can distinguish two different cases. In the first case standard model particles are entities that cannot exist independent of the brane. Well-known examples are goldstone modes of broken translational invariance (elastic waves of the brane), modes of open strings with end points stuck on the brane, or simply fermionic zero modes on the soliton-like wall. In this case, particles feel force but they cannot escape to the bulk. Another possibility is that the standard model particles are entities that exist independent of the brane. Then, I have to introduce a localizing potential ΔW that keeps them on the brane. Since on colliders we do not see events in which particles just disappear, the depth of the localizing potential ΔW would have to be bigger than $\sim$1 TeV. From the present bound on the size of universal extra dimensions one knows that the range of the localizing potential should be less than 300 GeV [11]. Localizing potential can be due to short range (contact) interactions with the matter of the first type, or the brane itself. For phenomenologically acceptable energy densities on the brane, the gravitational attraction cannot provide localizing potential. The particle localized on the brane will feel an effective potential which is a combination of short distance localizing potential and repulsive potential. With the potential of depth ΔW, all particles lighter than ΔW will be in stable equilibrium on the brane. Particles heavier than ΔW would be in a metastable state on the brane, because their self-energy in the bulk is roughly their mass [Eq. (3.8)]. Metastable particles can then tunnel through the barrier into the bulk. The brane can, in principle, be populated with both types of particles. Intrinsically brane particles would be stable (with respect to escape to the bulk decay). Particles trapped on the brane, on the other hand, can decay by escape to the bulk. Upon leaving the brane those particles would gain energy of the order of their mass (bulk self-energy) in the vicinity of the brane (distance $\sim 1/M_*$). The recoil effect of the brane would produce stable particles (goldstone modes, zero modes) with a total energy of the order of the mass of the particle that escaped to the bulk. This kind of decay to the bulk would make a missing energy signal on colliders smaller than one expected in a scenario with an ordinary brane.

So far, the discussion has referred to a brane of infinitesimal thickness (delta function type brane). Real, physical branes have finite thickness. It would be interesting to see how the finite thickness affects particle dynamics and if it can provide a localization mechanism. Arguments that we used in the derivation of the repulsive force in the 5D model apply equally well to branes in space with more than one extra dimension. To completely understand particle dynamics, one would certainly like to have an exact relativistic solution.

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APPENDIX: INCOMPLETE GAMMA FUNCTION

The incomplete gamma function $\Gamma_\alpha(z)$ is defined as

$$\Gamma_\alpha(z)=\int_z^\infty e^{-t}t^{\alpha-1}dt. \tag{A1}$$

It satisfies

$$\frac{d}{dz}\Gamma_0(z)=-\frac{e^{-z}}{z}, \quad \int \Gamma_0(z)dz=-e^{-z}+z\Gamma_0(z). \tag{A2}$$

For small and large values of argument it has following expansions:

$$\Gamma_0(z)\approx-\gamma-\ln z+z+\mathcal{O}(z^2), \quad z\to 0;$$
$$\Gamma_0(z)\approx e^{-z}[1/z-1/z^2+2/z^3+\mathcal{O}(1/z^4)], \quad z\to\infty. \tag{A3}$$

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